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Reliability Testing

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Introduction:

Reliability tests are intended to find whether a system can operate satisfactorily for a specified period of time under prescribed operating conditions. Different types of reliability tests are conducted at various stages of the life cycle of a system as indicated in the following list:

Tests conducted during the design stage:

At the end of the design stage, prototypes are built and tested to analyze the failure modes and reliability of the system. The results of the tests are used to check whether the system is behaving as intended. The results are also used to modify or redesign the system for improved reliability.

Tests conducted during the construction (or manufacturing) stage:

Before the system is put into service, qualification and acceptance tests are conducted to prove that the design standards of reliability are met. In acceptance tests, the system are component is tested to determine whether it should be accepted or rejected (on an individual unit or lot basis). Based on the results, better quality control methods can be used to reduce the defects in construction or manufacture. In qualification tests, the system is tested to determine whether it truly qualifies for its intended application.

Tests conducted during the operating stage:

Tests are conducted during the operating stage of the system to find the failure rate and reliability of the system. These results are used to verify the reliability analysis conducted previously and also to find whether any modifications are needed in the design and/or operating procedures to improve the reliability.

The life of a system can be divided into three phases, based on the failure rate, as follows;

1. Infant-mortality phase (also known as burn-in phase) during which a high failure rate is observed due to manufacturing defects.
2. Operating-life phase during which a constant failure rate is observed due to random failures.
3. Aging or wear-out phase during which a high failure rate is observed due to mechanical wear of the components.

Objectives of Reliability Tests

Depending on the objectives, reliability tests can be classisfied into four categories as follows:

1. Longevity tests. The objective is to find the length of the useful life of the system has a constant failure rate.
2. MTBF tests: The objective is to determine the mean time between failures of the system.
3. Operating life tests. The objective is to find the ability of the system to perform without failures for a prescribed minimum time period
4. Reliability margin tests. The objective is to establish the margin of safety between the extreme operating conditions of the system.

In this paper, we study reliability test of individual failure data of 11 Kvfeederwith 10 autoreclosures. The failure times of ten autoreclosures are observed to be as follows.

i	0	1	2	3	4	5	6	7	8	9	10
Time(min)	0	5	8	25	34	35	39	40	44	50	57

Let the failure times of the components be arranged in an increasing order. The cumulative distribution function, F(t), represents the probability of realizing the failure time of the component less than or equal to t. Thus, F(ti) denotes the fraction of components failed in time $t_i$is $F(t_i) = \frac{i}{n}$

The reliability of the component at time t=ti. R(ti) can be expressed as

$R(t_i)=1-F(t_i)=\frac{n-i}{n}$

By using a forward-finite difference formula, the probability density function of the failure time can be expressed as

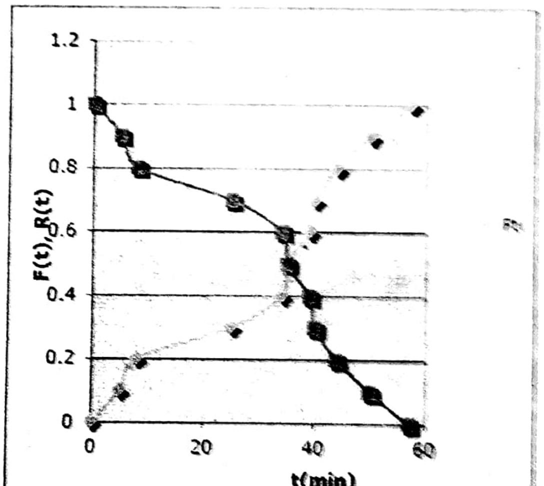
$$f(t) = \frac{F(ti+1)-F(ti)}{ti+1-ti} = \frac{1}{n(ti+1-ti)} \quad ; ti \le t \le ti+1 \; ; i = 1, 2, 3, \ldots\ldots n-1$$

The failure rate or hazard function of the component can be estimated as

$$h(t) = \frac{f(t)}{R(t)} = \frac{1}{(n-i)(ti+1-ti)} \quad ; ti \le t \le ti+1 \; ; i = 1, 2, 3, \ldots\ldots n-1$$

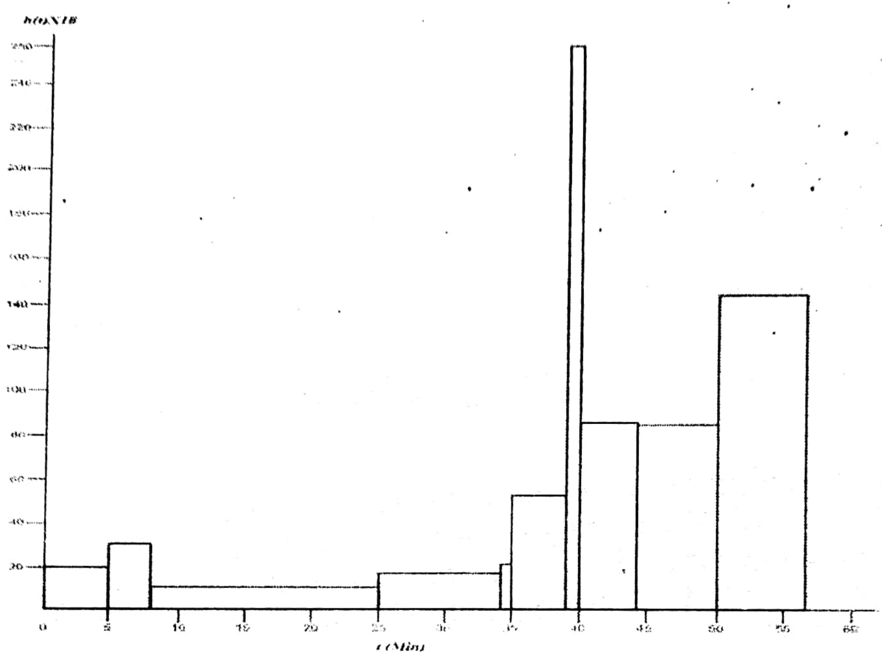
Computation of the hazard function from failure data

i	ti (min)	ti+1-ti	F(ti)	R(t)	f(t)	h(t)
0	0	5	0	1	.02	0.02
1	5	3	0.1	0.9	0.03	0.0333
2	8	17	0.2	0.8	0.00588	0.00735
3	25	9	0.3	0.7	0.0111	0.015857
4	34	1	0.4	0.6	0.1	0.1666
5	35	4	0.5	0.5	0.025	0.05
6	39	1	0.6	0.4	0.1	0.25
7	40	4	0.7	0.3	0.025	0.08333
8	44	6	0.8	0.2	0.0166	0.083
9	50	7	0.9	0.1	0.01428	0.1428
10	57	-	1	0	-	-



Variation of F(t) and R(t) with time(t)

Hazard function



Conclusion:

The length of 11 KV feeder, location of autoreclusures, type of faults, design and selection of switchgear equipment and performance of equipments plays an important role to improve the reliability of power system. The hazard function technics can be utilized to study the power system reliability.

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A Result Using Jordan Totient Function

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ABSTRACT *A real or complex valued function defined on the set of all positive integers is called an arithmetic function.An arithmetic function is said to be completely multiplicative function if f is not identically zero and f(mn) = f(m)f(n) for all m,n.Let k be a positiveinteger. In numbertheory, Jordan's totient function Jk (n) of a positive integer n is the number of k-tuples of positive integers all less than or equal to n that form a coprime (k+1) -tuple together with n. (A tuple is coprime if and only if it is coprime as a set.) This is a generalisation of Euler's totient function, which is J1.*

In this paper we present a review of an alternate proof for the generalizedBrauer - Rademacher identity.

Key Words: *: Arithmetic function, Multiplicative function, Jordan Totient function.*

I. Introduction

A real or complex valued function defined on the set of all positive integers is called an arithmetic function. An arithmetic function f is said to be multiplicative function in one argument if f is not identically zero and f(mn) = f(m)f(n) whenever (m,n) = 1. The function f(m,n) of two variables defined for pairs of positive integers m and n is said to be multiplicative in both the arguments m and n if f(1,1) = 1 and $f(m_1m_2,n_1n_2) = f(m_1,m_2)f(m_2,n_2)$ where $(m_1n_1,m_2n_2) = 1$.

An arithmetic function is said to be completely multiplicative function if f is not identically zero and f(mn) = f(m)f(n) for all m,n.

The Euler Totient function φ (n) is defined to be the number of positive integers not exceeding n which are relatively prime. The Mobius function μ (n) is defined by

$$(1.1)\quad \mu(n) = \begin{cases} 1 & if\ n = 1 \\ (-1)^k & if\ n = p_1p_2\cdots p_k, \\ 0 & otherwise \end{cases}$$

where p_i's are distinct primes.

Eckford cohen [3] introduced a function $\varphi_k(n)$, where $\varphi_k(n)$ denotes the number of non-negative integers less than N^k whichare relatively K-prime to N^k and

$$(1.2)\quad \sum_{d/n} \phi_k\left(N/d\right) = N^k$$

Nageswara rao [7] discussed the Jordan Totient function $J_k(N)$ which is defined for positive integers K and N to be the number of ordered sets of K non-negative integers less than N such that greatest common divisor of each set is prime to N and

$$(1.3)\quad \sum_{d/N} J_k\left(N/d\right) = N^k$$

By the Mobius Inversion formula, from (1.2) and (1.3) it follows that

$$(1.4)\quad J_k(N) = \phi_k(N) = \sum_{d/N} d^k \mu\left(\frac{N}{d}\right)$$

It is well known that [7, Theorem 6]

$$(1.5)\quad J_{k,l}(M) = M^{kl}\prod_{i=1}^{r}\left(1-\frac{1}{p_i^{kl}}\right),$$

where P_i's are distinct prime factors of M and $J_{k,l}(M)$ is the function defined as the number of ordered sets l integers less than M^k such that greatest common divisor of each set is relatively prime to M^k.

II. Preliminaries

The generalizedBrauer Rademacher identity is

$$(2.1)\quad J_{k,l}(M)\sum_{\substack{d/M \\ (d,N)=1}} \mu\left(\frac{M}{d}\right)\frac{d^{kl}}{J_{k,l}(d)} = \mu(M)\sum_{d/(M,N)} \mu\left(\frac{M}{d}\right) d^{kl}$$

Denote the left and right of (2.1) by $f_{kl}(M,N)$ and $g_{kl}(M,N)$ respectively.
We now prove the following lemmas.

2.2 Lemma: $J_{kl}(M)$ is a multiplicative function.

Proof: let M and M^1 be the positive integers such that $(M, M^1) = 1$

Consider $J_{k,l}(MM^1) = (MM^1)^{kl} \prod_i \left(1 - \frac{1}{r_i^{kl}}\right)$, where r_i's are distinct factors of MM^1

$$= M^{kl} M^{1kl} \prod_i \left(1 - \frac{1}{p_i^{kl}}\right) \prod_j \left(1 - \frac{1}{q_j^{kl}}\right)$$

where p_i's are distinct factors of M and q_j's are distinct factors of M^1

$$= M^{kl} \prod_i \left(1 - \frac{1}{p_i^{kl}}\right) M^{1kl} \prod_j \left(1 - \frac{1}{q_j^{kl}}\right)$$

$= J_{kl}(M)\, J_{kl}(M^1)$

Proving $J_{kl}(M)$ is a multiplicative function.

2.3 Lemma: $f_{kl}(M,N)$ is a multiplicative function in M and N.

Proof: Let M^1, N^1 be two numbers such that $(MN, M^1N^1) = 1$.

If d and d^1 are divisors of M and M^1 respectively and $dd^1 \mid MM^1$, then $\left(\frac{M}{d}, \frac{M^1}{d^1}\right) = 1$.

Consider $f_{kl}(M,N)\, f_{kl}(M^1,N^1)$

$$= \left\{ J_{k,l}(M) \sum_{\substack{d/M \\ (d,N)=1}} \mu\left(\frac{M}{d}\right) \frac{d^{kl}}{J_{k,l}(d)} \right\} \left\{ J_{k,l}(M^1) \sum_{\substack{d^1/M^1 \\ (d^1,N^1)=1}} \mu\left(\frac{M^1}{d^1}\right) \frac{d^{1kl}}{J_{k,l}(d^1)} \right\}$$

$$= J_{k,l}(M)\, J_{k,l}(M^1) \sum_{\substack{d/M \\ (d,N)=1}} \sum_{\substack{d^1/M^1 \\ (d^1,N^1)=1}} \mu\left(\frac{M}{d}\right) \mu\left(\frac{M^1}{d^1}\right) \frac{d^{kl} d^{1kl}}{J_{k,l}(d) J_{k,l}(d^1)}$$

$$= J_{k,l}(MM^1) \sum_{\substack{dd^1/MM^1 \\ (dd^1,NN^1)=1}} \mu\left(\frac{MM^1}{dd^1}\right) \frac{(dd^1)^{kl}}{J_{k,l}(dd^1)}$$

$= f_{kl}(MM^1, NN^1)$, proving $f_{kl}(M,N)$ is a multiplicative function.

2.4 Lemma: $g_{kl}(M,N)$ is a multiplicative function in M and N.

Proof: Let M^1, N^1 be two numbers such that $(MN, M^1N^1) = 1$.

Consider $g_{kl}(M,N)\, g_{kl}(M^1,N^1) =$

$$\left\{ \mu(M) \sum_{d/(M,N)} \mu\left(\frac{M}{d}\right) d^{kl} \right\} \left\{ \mu(M^1) \sum_{d^1/(M^1,N^1)} \mu\left(\frac{M^1}{d^1}\right) d^{1kl} \right\}$$

$$= \mu(M)\mu(M^1) \sum_{d/(M,N)} \sum_{d^1/(M^1,N^1)} \mu\left(\frac{M}{d}\right) \mu\left(\frac{M^1}{d^1}\right) (dd^1)^{kl}$$

$$= \mu(MM^1) \sum_{dd^1/(MM^1,NN^1)} \mu\left(\frac{MM^1}{dd^1}\right) (dd^1)^{kl}$$

$= g_{kl}(MM^1, NN^1)$, proving $g_{kl}(M,N)$ is a multiplicative function in M and N.

III. Main Result

The theorem below will give an alternate proof of the generalisedBrauer - Rademacher identity.

3.1Theorem: $f_{kl}(M,N) = g_{kl}(M,N)$.

Proof: In view of lemma 2.3 and lemma 2.4, it is sufficient to show that
$f_{kl}(M,N) = g_{kl}(M,N)$ when M and N are powers of a prime.
Let $M = p^m$ and $N = P^n$, Then

$$g_{kl}(p^m,p^n) = \mu(p) \sum_{d/(p,p^n)} \mu\left(\frac{p}{d}\right) d^{kl}$$

$$= (-1) \sum_{d/p} \mu\left(\frac{p}{d}\right) d^{kl}$$

$$(-1)\left[\mu\left(\frac{p}{p}\right) p^{kl} + \mu(p)\, 1^{kl}\right]$$

$= (-1)\,[p^{kl} - 1] = 1 - p^{kl}$

Case(iv): $m > 1$, $n \geq 0$ gives

$$g_{kl}(p^m,p^n) = \mu(p^m) \sum_{d/(p^m,p^n)} \mu\left(\frac{p^m}{d}\right) d^{kl}$$

$= 0$, Since $\mu(p^{\alpha}) = 0$ for all $\alpha \geq 1$

From the above cases, we get

$$(3.1.2) \qquad g_{kl}(p^m,p^n) = \begin{cases} 1 & \text{if } m=0, n\geq 0 \text{ and } m=1, n=0 \\ 1-p^{kl} & \text{if } m=1, n>0 \\ 0 & \text{if } m>1, n\geq 0 \end{cases}$$

From (3.1.1) and (3.1.2) it follows that

$$f_{kl}(p^m,p^n) = g_{kl}(p^m,p^n)$$

Therefore,

$$J_{k,l}(M) \sum_{\substack{d/M \\ (d,N)=1}} \mu\left(\frac{M}{d}\right) \frac{d^{kl}}{J_{k,l}(d)} = \mu(M) \sum_{d/(M,N)} \mu\left(\frac{M}{d}\right) d^{kl}$$

Which is generalizedBrauer – Rademacher identity.

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Results Using Relatively K – Prime Integers

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Abstract

An Arithmetic function is a real or complex valued function defined on the set of all positive integers.Consideringthese Arithmetic functions and their multiplicative property, we shall use relatively K – Prime integers and prove few useful identities.

Keywords: Arithmetic function, multiplicative function, K – prime integers

I. Introduction

The Euler Totient function φ (n) is defined to be the number of positive integers not exceeding n which are relatively prime to n .

(1.1) $\boldsymbol{\emptyset(n) = \sum_{\substack{d/n \\ (d,n)=1}} 1}$

An arithmetic function f is said to be multiplicative function in one argument if f is not identically zero and f(mn) = f(m)f(n) whenever (m,n) = 1. An arithmetic function is said to be completely multiplicative function if f is not identically zero and f(mn) = f(m)f(n) for all m,n.

If a and b are integers, not both zero, and k is any integer greater than 1, then $(a,b)_k$ denotes the largest common divisor of a and b which is also a k^{th} power. This will be referred to as k^{th} power greatest common divisor of a and b.

1.2 Definition: If$(a,b)_k$= 1, then a is said to be***relatively K – Prime*** to b.

1.3 EcfordCohen [3] introduced a function $\emptyset_k(\boldsymbol{n})$whichdenotes the number of non negative integers less than N^k which are relatively K-Prime to N^k.

(1.4)$\boldsymbol{\sum_{d/n} \emptyset_k(N/d) = N^k}$

By the mobius inversion formula, we get that

(1.5)$\boldsymbol{\emptyset_k(N) = \sum_{d/N} d^k \mu(N/d)}$

II. Preliminaries

2.1 lemma: $\phi_k(r)$ is multiplicative function.

Proof: In view of (1.5), we get

$$\phi_k(r) = \sum_{(d|r)} d^k \mu\left(\frac{r}{d}\right)$$

In order to prove multiplicative, we have to show that

$\phi_k(r_1r_2) = \phi_k(r_1)\phi_k(r_2)$ whenever $(r_1r_2) = 1$

Suppose that$r_1, r_2$are positive integers such that $(r_1r_2) = 1$. Then

(2.2)$\phi_k(r_1r_2) = \sum_{(d|r_1r_2)} d^k\mu\left(\frac{r_1r_2}{d}\right)$

Every divisor d ofr_1r_2 can beuniquely written as d = d_1d_2 where d_1/r_1 *and* d_2/r_2

Then (2.2) can be written as

$$\phi_k(r_1r_2) = \sum_{(d_1d_2|r_1r_2)} (d_1d_2)^k\mu\left(\frac{r_1r_2}{d_1d_2}\right)$$

$$= \sum_{(d_1|r_1)}\sum_{(d_2|r_2)} d_1^kd_2^k\,\mu\left(\frac{r_1}{d_1}\right)\mu\left(\frac{r_2}{d_2}\right)$$

$$= \left(\sum_{(d_1|r_1)} d_1^k\mu\left(\frac{r_1}{d_1}\right)\right)\left(\sum_{(d_2|r_2)} d_2^k\,\mu\left(\frac{r_2}{d_2}\right)\right)$$

$$= \phi_k(r_1)\phi_k(r_2)$$

Proving the lemma.

2.3 Lemma: Let$f(r) = \frac{r^k}{\phi_k(r)}$. Then $f(r)$ is strongly multiplicative function.

Proof: First we see that f is a multiplicative function.

That is,$f(r_1r_2) = f(r_1)f(r_2)$

Consider $f(r_1r_2) = \frac{(r_1r_2)^k}{\phi_k(r_1r_2)}$

$= \frac{r_1^kr_2^k}{(\phi_k(r_1)\phi_k(r_2))}$in view of lemma 2.1

$= \left(\frac{r_1^k}{\phi_k(r_1)}\right)\left(\frac{r_2^k}{\phi_k(r_2)}\right)$

$= f(r_1)f(r_2)$

Thus $f(r)$ is multiplicative function.

Also, we have for every prime p,

$$f(p) = \frac{p^k}{\phi_k(P)}$$

$$= \frac{p^k}{\sum_{\frac{d}{p}} d^k\mu\left(\frac{p}{d}\right)}$$

$$= \frac{P^k}{1^k\mu(P) + P^k\mu(1)}$$

$$= \frac{P^k}{P^k - 1}$$

and

$$f(P^2) = \frac{P^{2k}}{\phi_k(P^2)}$$

$$= \frac{P^{2k}}{\sum_{(d|P^2)} d^k\mu\left(\frac{P^2}{d}\right)}$$

$$= \frac{P^{2k}}{1^k\mu(P^2) + P^k\mu(P) + P^{2k}\mu(1)}$$

$$= \frac{P^{2k}}{-P^k + P^{2k}}$$

$$= \frac{P^k}{P^k - 1}$$

Similarly it can be shown that

$$f(P^3) = f(P^4) = \cdots = \frac{P^k}{P^k - 1}$$

Thus proving that $f(r)$ is strongly multiplication function.

III. Main Result

3.1 Theorem: Let $g(r) = \frac{1}{\phi_k(r)}$. Then $\frac{P^k}{\phi_k(P)} - 1 = \frac{1}{\phi_k(p)}$

Proof: Consider $g(p) = \frac{1}{\phi_k(p)}$

$$= \frac{1}{\sum_{\frac{d}{p}} d^k\mu\left(\frac{P}{d}\right)}, \text{ from } (1.5)$$

$$= \frac{1}{1^k\mu(P) + P^k\mu(1)}$$

$$= \frac{1}{P^k - 1}$$

and consider $\frac{P^k}{\phi_k(P)} - 1$

$$= \frac{P^k}{P^k - 1} - 1$$

$$= \frac{1}{P^k - 1}$$

Thus proving the result.

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3.2 Theorem: $$\sum_{(d|r)} \frac{d^k}{\phi_k(d)}\, \mu\left(\frac{r}{d}\right) = \mu(r)\, \mu\left(\frac{r}{t}\right) \frac{1}{\phi_k\left(\frac{r}{t}\right)}$$

Proof: In view of lemma 2.3, we have $f(r) = \frac{r^k}{\phi_k(r)}$ which gives $f(d) = \frac{d^k}{\phi_k(d)}$

and ([5], Theorem 3.1) gives for $n \geq 1, m \geq 1$, we have

$$(3.3) \sum_{\substack{d|n \\ (d,m)=1}} f(d)\mu\left(\frac{n}{d}\right) = \mu(n)\mu\left(\frac{n}{t}\right) h\left(\frac{n}{t}\right), \text{ where } t = (n, m)$$

using (3.3), $\sum_{(d|r)} \frac{d^k}{\phi_k(d)}\, \mu\left(\frac{r}{d}\right) = \mu(r)\mu\left(\frac{r}{t}\right) h\left(\frac{n}{t}\right)$

$= \mu(r)\mu\left(\frac{r}{t}\right) \frac{1}{\phi_k\left(\frac{r}{t}\right)}$, from theorem 3.1.

Hence the result.

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